R-R Chain Design in Modified Yu's Coordinates

Insert Library Functions

```
> with(dualvectors):
Share Library:  dualvectors
Author: Perez,Alba.
Description:  Performs dual vectors and numbers operations.
> with(quaternion):
Share Library:  quaternion
Author: Perez,Alba.
Description:  Performs quaternion operations.
> with(linalg):
Warning, new definition for norm
Warning, new definition for trace
```

Insert data a,b,c,kappa (No numeric values yet)

```
> a:='a'; b:='b'; c:='c'; kappa:='kappa';
```

$$a := a$$
$$b := b$$
$$c := c$$
$$\kappa := \kappa$$

Yu's Coordinates for a Bennett Linkage. Expression of G,W

```
> Apoint:=vector(3, [a*cos(kappa/2), a*sin(kappa/2), -c/2]);
```

$$Apoint := \left[a \cos\left(\frac{1}{2}\kappa\right), a \sin\left(\frac{1}{2}\kappa\right), -\frac{1}{2}c \right]$$

```
> Bpoint:=vector(3, [b*cos(kappa/2), -b*sin(kappa/2), c/2]);
```

$$Bpoint := \left[b \cos\left(\frac{1}{2}\kappa\right), -b \sin\left(\frac{1}{2}\kappa\right), \frac{1}{2}c \right]$$

```
> Cpoint:=vector(3, [-b*cos(kappa/2), b*sin(kappa/2), c/2]);
```

$$Cpoint := \left[-b \cos\left(\frac{1}{2}\kappa\right), b \sin\left(\frac{1}{2}\kappa\right), \frac{1}{2}c \right]$$

```
> Dpoint:=vector(3, [-a*cos(kappa/2), -a*sin(kappa/2), -c/2]);
```

$$Dpoint := \left[-a \cos\left(\frac{1}{2}\kappa\right), -a \sin\left(\frac{1}{2}\kappa\right), -\frac{1}{2}c \right]$$

```
> AC:=evalm(Cpoint-Apoint); AB:=evalm(Bpoint-Apoint);
```

$$AC := \left[-b \cos\left(\frac{1}{2}\kappa\right) - a \cos\left(\frac{1}{2}\kappa\right), b \sin\left(\frac{1}{2}\kappa\right) - a \sin\left(\frac{1}{2}\kappa\right), c \right]$$

$$AB := \left[b \cos\left(\frac{1}{2}\kappa\right) - a \cos\left(\frac{1}{2}\kappa\right), -b \sin\left(\frac{1}{2}\kappa\right) - a \sin\left(\frac{1}{2}\kappa\right), c \right]$$

```
> BA:=evalm(-AB);BD:=evalm(Dpoint-Bpoint);
```

$$BA := \left[-b \cos\left(\frac{1}{2}\kappa\right) + a \cos\left(\frac{1}{2}\kappa\right), b \sin\left(\frac{1}{2}\kappa\right) + a \sin\left(\frac{1}{2}\kappa\right), -c \right]$$

$$BD := \left[-b \cos\left(\frac{1}{2}\kappa\right) - a \cos\left(\frac{1}{2}\kappa\right), b \sin\left(\frac{1}{2}\kappa\right) - a \sin\left(\frac{1}{2}\kappa\right), -c \right]$$

```
> CA:=evalm(-AC);CD:=evalm(Dpoint-Cpoint);
```

$$CA := \left[b \cos\left(\frac{1}{2}\kappa\right) + a \cos\left(\frac{1}{2}\kappa\right), -b \sin\left(\frac{1}{2}\kappa\right) + a \sin\left(\frac{1}{2}\kappa\right), -c \right]$$

$$CD := \left[b \cos\left(\frac{1}{2}\kappa\right) - a \cos\left(\frac{1}{2}\kappa\right), -b \sin\left(\frac{1}{2}\kappa\right) - a \sin\left(\frac{1}{2}\kappa\right), -c \right]$$

```
> DB:=evalm(-BD); DC:=evalm(-CD);
```

$$DB := \left[b\cos\left(\frac{1}{2}\kappa\right) + a\cos\left(\frac{1}{2}\kappa\right), -b\sin\left(\frac{1}{2}\kappa\right) + a\sin\left(\frac{1}{2}\kappa\right), c \right]$$

$$DC := \left[-b\cos\left(\frac{1}{2}\kappa\right) + a\cos\left(\frac{1}{2}\kappa\right), b\sin\left(\frac{1}{2}\kappa\right) + a\sin\left(\frac{1}{2}\kappa\right), c \right]$$

>

>

G and W:

>

Compute Gline (fixed axis A)

>

```
> epsilon:='epsilon';
```

$$\varepsilon := \varepsilon$$

```
> Gtemp:=simplify(crossprod(AC,AB));
```

$$Gtemp := \left[2\,c\,b\sin\left(\frac{1}{2}\kappa\right), 2\,c\,b\cos\left(\frac{1}{2}\kappa\right), 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right) \right]$$

```
> Gmag:=simplify(sqrt(dotprod(Gtemp, Gtemp, 'orthogonal')));
```

$$Gmag := 2\sqrt{-b^2\left(-c^2 - 4\cos\left(\frac{1}{2}\kappa\right)^2 a^2 + 4\cos\left(\frac{1}{2}\kappa\right)^4 a^2\right)}$$

```
> Gdir:=evalm(Gtemp);
```

$$Gdir := \left[2\,c\,b\sin\left(\frac{1}{2}\kappa\right), 2\,c\,b\cos\left(\frac{1}{2}\kappa\right), 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right) \right]$$

```
> Gmom:=simplify(crossprod(Apoint,Gdir));
```

$$Gmom := \left[4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right) - 4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right)^3 + c^2\,b\cos\left(\frac{1}{2}\kappa\right), -c^2\,b\sin\left(\frac{1}{2}\kappa\right) - 4\,a^2\cos\left(\frac{1}{2}\kappa\right)^2 b\sin\left(\frac{1}{2}\kappa\right), 4\,a\cos\left(\frac{1}{2}\kappa\right)^2 c\,b - 2\,a\,c\,b \right]$$

```
> Gscrewep:=evalm(Gdir+epsilon*Gmom);
```

$$Gscrewep := \left[2\,c\,b\sin\left(\frac{1}{2}\kappa\right) + \varepsilon\left(4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right) - 4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right)^3 + c^2\,b\cos\left(\frac{1}{2}\kappa\right)\right), 2\,c\,b\cos\left(\frac{1}{2}\kappa\right) + \varepsilon\left(-c^2\,b\sin\left(\frac{1}{2}\kappa\right) - 4\,a^2\cos\left(\frac{1}{2}\kappa\right)^2 b\sin\left(\frac{1}{2}\kappa\right)\right), 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right) + \varepsilon\left(4\,a\cos\left(\frac{1}{2}\kappa\right)^2 c\,b - 2\,a\,c\,b\right) \right]$$

```
> Gscrew:=concat(Gdir, Gmom);
```

$$Gscrew := \begin{bmatrix} 2\,c\,b\sin\left(\frac{1}{2}\kappa\right) & 4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right) - 4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right)^3 + c^2\,b\cos\left(\frac{1}{2}\kappa\right) \\ 2\,c\,b\cos\left(\frac{1}{2}\kappa\right) & -c^2\,b\sin\left(\frac{1}{2}\kappa\right) - 4\,a^2\cos\left(\frac{1}{2}\kappa\right)^2 b\sin\left(\frac{1}{2}\kappa\right) \\ 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right) & 4\,a\cos\left(\frac{1}{2}\kappa\right)^2 c\,b - 2\,a\,c\,b \end{bmatrix}$$

```
> Gline:=concat(Gdir/Gmag,Gmom/Gmag);
```

$$Gline :=$$

$$\begin{bmatrix} \dfrac{c\,b\sin\left(\frac{1}{2}\kappa\right)}{\sqrt{-b^2\left(-c^2-4\cos\left(\frac{1}{2}\kappa\right)^2a^2+4\cos\left(\frac{1}{2}\kappa\right)^4a^2\right)}}, & \dfrac{1}{2}\dfrac{4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right)-4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right)^3+c^2\,b\cos\left(\frac{1}{2}\kappa\right)}{\sqrt{-b^2\left(-c^2-4\cos\left(\frac{1}{2}\kappa\right)^2a^2+4\cos\left(\frac{1}{2}\kappa\right)^4a^2\right)}} \\ \dfrac{c\,b\cos\left(\frac{1}{2}\kappa\right)}{\sqrt{-b^2\left(-c^2-4\cos\left(\frac{1}{2}\kappa\right)^2a^2+4\cos\left(\frac{1}{2}\kappa\right)^4a^2\right)}}, & \dfrac{1}{2}\dfrac{-c^2\,b\sin\left(\frac{1}{2}\kappa\right)-4\,a^2\cos\left(\frac{1}{2}\kappa\right)^2b\sin\left(\frac{1}{2}\kappa\right)}{\sqrt{-b^2\left(-c^2-4\cos\left(\frac{1}{2}\kappa\right)^2a^2+4\cos\left(\frac{1}{2}\kappa\right)^4a^2\right)}} \\ 2\dfrac{b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right)}{\sqrt{-b^2\left(-c^2-4\cos\left(\frac{1}{2}\kappa\right)^2a^2+4\cos\left(\frac{1}{2}\kappa\right)^4a^2\right)}}, & \dfrac{1}{2}\dfrac{4\,a\cos\left(\frac{1}{2}\kappa\right)^2c\,b-2\,a\,c\,b}{\sqrt{-b^2\left(-c^2-4\cos\left(\frac{1}{2}\kappa\right)^2a^2+4\cos\left(\frac{1}{2}\kappa\right)^4a^2\right)}} \end{bmatrix}$$

Compute Wline (moving axis B)

```
> Wtemp:=simplify(crossprod(BA, BD));
```

$$Wtemp := \left[-2\,c\,a\sin\left(\frac{1}{2}\kappa\right), 2\,c\,a\cos\left(\frac{1}{2}\kappa\right), 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right)\right]$$

```
> Wmag:=simplify(sqrt(dotprod(Wtemp, Wtemp, 'orthogonal')));
```

$$Wmag := 2\sqrt{-a^2\left(-c^2-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^4\right)}$$

```
> Wdir:=evalm(Wtemp);
```

$$Wdir := \left[-2\,c\,a\sin\left(\frac{1}{2}\kappa\right), 2\,c\,a\cos\left(\frac{1}{2}\kappa\right), 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right)\right]$$

```
> Wmom:=simplify(crossprod(Bpoint, Wdir));
```

$$Wmom := \left[-4\,b^2\cos\left(\frac{1}{2}\kappa\right)a+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^3a-c^2\,a\cos\left(\frac{1}{2}\kappa\right), -c^2\,a\sin\left(\frac{1}{2}\kappa\right)-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2a\sin\left(\frac{1}{2}\kappa\right), 4\,a\cos\left(\frac{1}{2}\kappa\right)^2c\,b-2\,a\,c\,b\right]$$

```
> Wscrewep:=evalm(Wdir+epsilon*Wmom);
```

$$Wscrewep := \left[-2\,c\,a\sin\left(\frac{1}{2}\kappa\right)+\varepsilon\left(-4\,b^2\cos\left(\frac{1}{2}\kappa\right)a+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^3a-c^2\,a\cos\left(\frac{1}{2}\kappa\right)\right), 2\,c\,a\cos\left(\frac{1}{2}\kappa\right)+\varepsilon\left(-c^2\,a\sin\left(\frac{1}{2}\kappa\right)-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2a\sin\left(\frac{1}{2}\kappa\right)\right), 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right)+\varepsilon\left(4\,a\cos\left(\frac{1}{2}\kappa\right)^2c\,b-2\,a\,c\,b\right)\right]$$

```
Wscrew:=concat(Wdir,Wmom);
```

$$Wscrew := \begin{bmatrix} -2\,c\,a\sin\left(\frac{1}{2}\kappa\right) & -4\,b^2\cos\left(\frac{1}{2}\kappa\right)a+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^3a-c^2\,a\cos\left(\frac{1}{2}\kappa\right) \\ 2\,c\,a\cos\left(\frac{1}{2}\kappa\right) & -c^2\,a\sin\left(\frac{1}{2}\kappa\right)-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2a\sin\left(\frac{1}{2}\kappa\right) \\ 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right) & 4\,a\cos\left(\frac{1}{2}\kappa\right)^2c\,b-2\,a\,c\,b \end{bmatrix}$$

```
> Wline:=concat(Wdir/Wmag,Wmom/Wmag);
```

$$Wline := \left[-\frac{c\,a\sin\left(\frac{1}{2}\kappa\right)}{\sqrt{-a^2\left(-c^2-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^4\right)}},\right.$$

$$\frac{1}{2}\frac{-4\,b^2\cos\left(\frac{1}{2}\kappa\right)a+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^3a-c^2\,a\cos\left(\frac{1}{2}\kappa\right)}{\sqrt{-a^2\left(-c^2-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^4\right)}}\Bigg]$$

$$\left[\frac{c\,a\cos\left(\frac{1}{2}\kappa\right)}{\sqrt{-a^2\left(-c^2-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^4\right)}},\frac{1}{2}\frac{-c^2\,a\sin\left(\frac{1}{2}\kappa\right)-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2a\sin\left(\frac{1}{2}\kappa\right)}{\sqrt{-a^2\left(-c^2-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^4\right)}}\right]$$

$$\left[2\frac{b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right)}{\sqrt{-a^2\left(-c^2-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^4\right)}},\frac{1}{2}\frac{4\,a\cos\left(\frac{1}{2}\kappa\right)^2c\,b-2\,a\,c\,b}{\sqrt{-a^2\left(-c^2-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^4\right)}}\right]$$

H and V:

Compute Hline (fixed axis C)

```
> Htemp:=simplify(crossprod(CD,CA));
```

$$Htemp := \left[2\,c\,a\sin\left(\frac{1}{2}\kappa\right), -2\,c\,a\cos\left(\frac{1}{2}\kappa\right), 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right)\right]$$

```
> Hmag:=simplify(sqrt(dotprod(Htemp, Htemp, 'orthogonal')));
```

$$Hmag := 2\sqrt{-a^2\left(-c^2-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^4\right)}$$

```
> Hdir:=evalm(Htemp);
```

$$Hdir := \left[2\,c\,a\sin\left(\frac{1}{2}\kappa\right), -2\,c\,a\cos\left(\frac{1}{2}\kappa\right), 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right)\right]$$

```
> Hmom:=simplify(crossprod(Cpoint,Hdir));
```

$$Hmom := \left[4\,b^2\cos\left(\frac{1}{2}\kappa\right)a-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^3a+c^2\,a\cos\left(\frac{1}{2}\kappa\right), c^2\,a\sin\left(\frac{1}{2}\kappa\right)+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2a\sin\left(\frac{1}{2}\kappa\right), 4\,a\cos\left(\frac{1}{2}\kappa\right)^2c\,b-2\,a\,c\,b\right]$$

```
> Hscrewep:=evalm(Hdir+epsilon*Hmom);
```

$$Hscrewep := \left[2\,c\,a\sin\left(\frac{1}{2}\kappa\right)+\varepsilon\left(4\,b^2\cos\left(\frac{1}{2}\kappa\right)a-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^3a+c^2\,a\cos\left(\frac{1}{2}\kappa\right)\right), -2\,c\,a\cos\left(\frac{1}{2}\kappa\right)+\varepsilon\left(c^2\,a\sin\left(\frac{1}{2}\kappa\right)+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2a\sin\left(\frac{1}{2}\kappa\right)\right), 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right)+\varepsilon\left(4\,a\cos\left(\frac{1}{2}\kappa\right)^2c\,b-2\,a\,c\,b\right)\right]$$

```
> Hscrew:=concat(Hdir, Hmom);
```

$$Hscrew := \begin{bmatrix} 2\,c\,a\sin\left(\frac{1}{2}\kappa\right) & 4\,b^2\cos\left(\frac{1}{2}\kappa\right)a-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^3a+c^2\,a\cos\left(\frac{1}{2}\kappa\right) \\ -2\,c\,a\cos\left(\frac{1}{2}\kappa\right) & c^2\,a\sin\left(\frac{1}{2}\kappa\right)+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2a\sin\left(\frac{1}{2}\kappa\right) \\ 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right) & 4\,a\cos\left(\frac{1}{2}\kappa\right)^2c\,b-2\,a\,c\,b \end{bmatrix}$$

```
> Hline:=concat(Hdir/Hmag,Hmom/Hmag);
```

$$Hline :=$$

$$\begin{bmatrix} \dfrac{c\,a\,\sin\left(\frac{1}{2}\kappa\right)}{\sqrt{-a^2\left(-c^2-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^4\right)}}, & \frac{1}{2}\dfrac{4\,b^2\cos\left(\frac{1}{2}\kappa\right)a-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^3a+c^2\,a\cos\left(\frac{1}{2}\kappa\right)}{\sqrt{-a^2\left(-c^2-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^4\right)}} \\ -\dfrac{c\,a\cos\left(\frac{1}{2}\kappa\right)}{\sqrt{-a^2\left(-c^2-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^4\right)}}, & \frac{1}{2}\dfrac{c^2\,a\sin\left(\frac{1}{2}\kappa\right)+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2a\sin\left(\frac{1}{2}\kappa\right)}{\sqrt{-a^2\left(-c^2-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^4\right)}} \\ 2\dfrac{b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right)}{\sqrt{-a^2\left(-c^2-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^4\right)}}, & \frac{1}{2}\dfrac{4\,a\cos\left(\frac{1}{2}\kappa\right)^2c\,b-2\,a\,c\,b}{\sqrt{-a^2\left(-c^2-4\,b^2\cos\left(\frac{1}{2}\kappa\right)^2+4\,b^2\cos\left(\frac{1}{2}\kappa\right)^4\right)}} \end{bmatrix}$$

Compute Vline (moving axis D)

```
> Vtemp:=simplify(crossprod(DB, DC));
```

$$Vtemp := \left[-2\,c\,b\sin\left(\frac{1}{2}\kappa\right), -2\,c\,b\cos\left(\frac{1}{2}\kappa\right), 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right)\right]$$

```
> Vmag:=simplify(sqrt(dotprod(Vtemp, Vtemp, 'orthogonal')));
```

$$Vmag := 2\sqrt{-b^2\left(-c^2-4\cos\left(\frac{1}{2}\kappa\right)^2a^2+4\cos\left(\frac{1}{2}\kappa\right)^4a^2\right)}$$

```
> Vdir:=evalm(Vtemp);
```

$$Vdir := \left[-2\,c\,b\sin\left(\frac{1}{2}\kappa\right), -2\,c\,b\cos\left(\frac{1}{2}\kappa\right), 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right)\right]$$

```
> Vmom:=simplify(crossprod(Dpoint,Vdir));
```

$$Vmom := \left[-4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right)+4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right)^3-c^2\,b\cos\left(\frac{1}{2}\kappa\right), c^2\,b\sin\left(\frac{1}{2}\kappa\right)+4\,a^2\cos\left(\frac{1}{2}\kappa\right)^2b\sin\left(\frac{1}{2}\kappa\right), 4\,a\cos\left(\frac{1}{2}\kappa\right)^2c\,b-2\,a\,c\,b\right]$$

```
> Vscrewep:=evalm(Vdir+epsilon*Vmom);
```

$$Vscrewep := \left[-2\,c\,b\sin\left(\frac{1}{2}\kappa\right)+\varepsilon\left(-4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right)+4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right)^3-c^2\,b\cos\left(\frac{1}{2}\kappa\right)\right), -2\,c\,b\cos\left(\frac{1}{2}\kappa\right)+\varepsilon\left(c^2\,b\sin\left(\frac{1}{2}\kappa\right)+4\,a^2\cos\left(\frac{1}{2}\kappa\right)^2b\sin\left(\frac{1}{2}\kappa\right)\right), 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right)+\varepsilon\left(4\,a\cos\left(\frac{1}{2}\kappa\right)^2c\,b-2\,a\,c\,b\right)\right]$$

```
> Vscrew:=concat(Vdir, Vmom);
```

$$Vscrew := \begin{bmatrix} -2\,c\,b\sin\left(\frac{1}{2}\kappa\right) & -4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right)+4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right)^3-c^2\,b\cos\left(\frac{1}{2}\kappa\right) \\ -2\,c\,b\cos\left(\frac{1}{2}\kappa\right) & c^2\,b\sin\left(\frac{1}{2}\kappa\right)+4\,a^2\cos\left(\frac{1}{2}\kappa\right)^2b\sin\left(\frac{1}{2}\kappa\right) \\ 4\,b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right) & 4\,a\cos\left(\frac{1}{2}\kappa\right)^2c\,b-2\,a\,c\,b \end{bmatrix}$$

```
> Vline:=concat(Vdir/Vmag,Vmom/Vmag);
```

$$Vline := \left[-\frac{c\,b\sin\left(\frac{1}{2}\kappa\right)}{\sqrt{-b^2\left(-c^2-4\cos\left(\frac{1}{2}\kappa\right)^2a^2+4\cos\left(\frac{1}{2}\kappa\right)^4a^2\right)}}, \frac{1}{2}\frac{-4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right)+4\,a^2\,b\cos\left(\frac{1}{2}\kappa\right)^3-c^2\,b\cos\left(\frac{1}{2}\kappa\right)}{\sqrt{-b^2\left(-c^2-4\cos\left(\frac{1}{2}\kappa\right)^2a^2+4\cos\left(\frac{1}{2}\kappa\right)^4a^2\right)}}\right]$$

$$\left[-\frac{c\,b\cos\left(\frac{1}{2}\kappa\right)}{\sqrt{-b^2\left(-c^2-4\cos\left(\frac{1}{2}\kappa\right)^2a^2+4\cos\left(\frac{1}{2}\kappa\right)^4a^2\right)}},\frac{1}{2}\frac{c^2\,b\sin\left(\frac{1}{2}\kappa\right)+4\,a^2\cos\left(\frac{1}{2}\kappa\right)^2b\sin\left(\frac{1}{2}\kappa\right)}{\sqrt{-b^2\left(-c^2-4\cos\left(\frac{1}{2}\kappa\right)^2a^2+4\cos\left(\frac{1}{2}\kappa\right)^4a^2\right)}}\right]$$

$$\left[2\frac{b\cos\left(\frac{1}{2}\kappa\right)a\sin\left(\frac{1}{2}\kappa\right)}{\sqrt{-b^2\left(-c^2-4\cos\left(\frac{1}{2}\kappa\right)^2a^2+4\cos\left(\frac{1}{2}\kappa\right)^4a^2\right)}},\frac{1}{2}\frac{4\,a\cos\left(\frac{1}{2}\kappa\right)^2c\,b-2\,a\,c\,b}{\sqrt{-b^2\left(-c^2-4\cos\left(\frac{1}{2}\kappa\right)^2a^2+4\cos\left(\frac{1}{2}\kappa\right)^4a^2\right)}}\right]$$

```
>
```

Insert screws S12, S13 (No numeric values yet)

Define Screws A=S12 and B=S13

```
> delta1:='delta1'; d1:='d1';delta2:='delta2'; d2:='d2';
```

$$\delta 1 := \delta 1$$
$$d1 := d1$$
$$\delta 2 := \delta 2$$
$$d2 := d2$$

```
> Aline:=matrix(3,2, [cos(delta1),-d1*sin(delta1),
  sin(delta1),d1*cos(delta1),0,0]);
```

$$Aline := \begin{bmatrix} \cos(\delta 1) & -d1\sin(\delta 1) \\ \sin(\delta 1) & d1\cos(\delta 1) \\ 0 & 0 \end{bmatrix}$$

```
> Adir:=col(Aline,1);
```

$$Adir := [\cos(\delta 1), \sin(\delta 1), 0]$$

```
> Bline:=matrix(3,2, [cos(delta2),-d2*sin(delta2),
  sin(delta2),d2*cos(delta2),0,0]);
```

$$Bline := \begin{bmatrix} \cos(\delta 2) & -d2\sin(\delta 2) \\ \sin(\delta 2) & d2\cos(\delta 2) \\ 0 & 0 \end{bmatrix}$$

```
> Bdir:=col(Bline,1);
```

$$Bdir := [\cos(\delta 2), \sin(\delta 2), 0]$$

```
> psi:='psi';y:='y';
```

$$\psi := \psi$$
$$y := y$$

```
> Amag:=sin(psi[1]/2); Apitch:=(y[1]/2)*cos(psi[1]/2)/sin(psi[1]/2);
  Bmag:=sin(psi[2]/2);Bpitch:=(y[2]/2)*cos(psi[2]/2)/sin(psi[2]/2);
```

$$Amag := \sin\left(\frac{1}{2}\psi_1\right)$$
$$Apitch := \frac{1}{2}\frac{y_1\cos\left(\frac{1}{2}\psi_1\right)}{\sin\left(\frac{1}{2}\psi_1\right)}$$
$$Bmag := \sin\left(\frac{1}{2}\psi_2\right)$$
$$Bpitch := \frac{1}{2}\frac{y_2\cos\left(\frac{1}{2}\psi_2\right)}{\sin\left(\frac{1}{2}\psi_2\right)}$$

```
> Ascrew:=concat(Amag*col(Aline,1),
  Amag*col(Aline,2)+Amag*Apitch*col(Aline,1));
```

$$Ascrew := \begin{bmatrix} \sin\left(\frac{1}{2}\psi_1\right)\cos(\delta 1) & -\sin\left(\frac{1}{2}\psi_1\right) d1 \sin(\delta 1) + \frac{1}{2} y_1 \cos\left(\frac{1}{2}\psi_1\right)\cos(\delta 1) \\ \sin\left(\frac{1}{2}\psi_1\right)\sin(\delta 1) & \sin\left(\frac{1}{2}\psi_1\right) d1 \cos(\delta 1) + \frac{1}{2} y_1 \cos\left(\frac{1}{2}\psi_1\right)\sin(\delta 1) \\ 0 & 0 \end{bmatrix}$$

```
> Bscrew:=concat(Bmag*col(Bline,1),
  Bmag*col(Bline,2)+Bmag*Bpitch*col(Bline,1));
```

$$Bscrew := \begin{bmatrix} \sin\left(\frac{1}{2}\psi_2\right)\cos(\delta 2) & -\sin\left(\frac{1}{2}\psi_2\right) d2 \sin(\delta 2) + \frac{1}{2} y_2 \cos\left(\frac{1}{2}\psi_2\right)\cos(\delta 2) \\ \sin\left(\frac{1}{2}\psi_2\right)\sin(\delta 2) & \sin\left(\frac{1}{2}\psi_2\right) d2 \cos(\delta 2) + \frac{1}{2} y_2 \cos\left(\frac{1}{2}\psi_2\right)\sin(\delta 2) \\ 0 & 0 \end{bmatrix}$$

```
>
```

Generate constraints

The direction constraint for S12:

```
> num:=simplify(dotprod(Gdir,crossprod(col(Aline,1),Wdir),orthogonal));
>
```

$$num := 8\,c\,b^2 \sin(\delta 1)\cos\left(\frac{1}{2}\kappa\right) a - 8\,c\,b^2 \sin(\delta 1)\cos\left(\frac{1}{2}\kappa\right)^3 a - 8\,c\,b^2 \cos\left(\frac{1}{2}\kappa\right)^2 \cos(\delta 1)\, a \sin\left(\frac{1}{2}\kappa\right) + 8\,b\cos\left(\frac{1}{2}\kappa\right)^2 a^2 \sin\left(\frac{1}{2}\kappa\right)\cos(\delta 1)\, c + 8\,b\cos\left(\frac{1}{2}\kappa\right) a^2 \sin(\delta 1)\, c - 8\,b\cos\left(\frac{1}{2}\kappa\right)^3 a^2 \sin(\delta 1)\, c$$

```
> den:=simplify(dotprod(crossprod(Gdir,col(Aline,1)),crossprod(col(Aline,1),Wdi
  r),orthogonal));
```

$$den := -16\,b^2 \cos\left(\frac{1}{2}\kappa\right)^2 a^2 + 16\,b^2 \cos\left(\frac{1}{2}\kappa\right)^4 a^2 + 4\,c^2\,b\,a - 4\,c^2\,b\,a\cos\left(\frac{1}{2}\kappa\right)^2 - 4\,c^2\,b\,a\cos(\delta 1)^2$$

```
> algeq1:=tan(theta12/2)+num/den;
```

$$algeq1 := \tan\left(\frac{1}{2}\theta 12\right) + \left(8\,c\,b^2 \sin(\delta 1)\cos\left(\frac{1}{2}\kappa\right) a - 8\,c\,b^2 \sin(\delta 1)\cos\left(\frac{1}{2}\kappa\right)^3 a - 8\,c\,b^2 \cos\left(\frac{1}{2}\kappa\right)^2 \cos(\delta 1)\, a \sin\left(\frac{1}{2}\kappa\right) + 8\,b\cos\left(\frac{1}{2}\kappa\right)^2 a^2 \sin\left(\frac{1}{2}\kappa\right)\cos(\delta 1)\, c + 8\,b\cos\left(\frac{1}{2}\kappa\right) a^2 \sin(\delta 1)\, c - 8\,b\cos\left(\frac{1}{2}\kappa\right)^3 a^2 \sin(\delta 1)\, c\right) \Big/ \left(-16\,b^2 \cos\left(\frac{1}{2}\kappa\right)^2 a^2 + 16\,b^2 \cos\left(\frac{1}{2}\kappa\right)^4 a^2 + 4\,c^2\,b\,a - 4\,c^2\,b\,a\cos\left(\frac{1}{2}\kappa\right)^2 - 4\,c^2\,b\,a\cos(\delta 1)^2\right)$$

The direction constraint for S13

```
> num:=simplify(dotprod(Gdir,crossprod(col(Bline,1),Wdir),orthogonal));
```

$$num := 8\,c\,b^2 \sin(\delta 2)\cos\left(\frac{1}{2}\kappa\right) a - 8\,c\,b^2 \sin(\delta 2)\cos\left(\frac{1}{2}\kappa\right)^3 a - 8\,c\,b^2 \cos\left(\frac{1}{2}\kappa\right)^2 \cos(\delta 2)\, a \sin\left(\frac{1}{2}\kappa\right) + 8\,b\cos\left(\frac{1}{2}\kappa\right)^2 a^2 \sin\left(\frac{1}{2}\kappa\right)\cos(\delta 2)\, c + 8\,b\cos\left(\frac{1}{2}\kappa\right) a^2 \sin(\delta 2)\, c - 8\,b\cos\left(\frac{1}{2}\kappa\right)^3 a^2 \sin(\delta 2)\, c$$

```
> den:=simplify(dotprod(crossprod(Gdir,col(Bline,1)),crossprod(col(Bline,1),Wdi
  r),orthogonal));
```

$$den := -16\,b^2 \cos\left(\frac{1}{2}\kappa\right)^2 a^2 + 16\,b^2 \cos\left(\frac{1}{2}\kappa\right)^4 a^2 + 4\,c^2\,b\,a - 4\,c^2\,b\,a\cos\left(\frac{1}{2}\kappa\right)^2 - 4\,c^2\,b\,a\cos(\delta 2)^2$$

```
> algeq2:=tan(theta13/2)+num/den;
```

$$algeq2 := \tan\left(\frac{1}{2}\theta 13\right) + \left(8\,c\,b^2 \sin(\delta 2)\cos\left(\frac{1}{2}\kappa\right) a - 8\,c\,b^2 \sin(\delta 2)\cos\left(\frac{1}{2}\kappa\right)^3 a - 8\,c\,b^2 \cos\left(\frac{1}{2}\kappa\right)^2 \cos(\delta 2)\, a \sin\left(\frac{1}{2}\kappa\right) + 8\,b\cos\left(\frac{1}{2}\kappa\right)^2 a^2 \sin\left(\frac{1}{2}\kappa\right)\cos(\delta 2)\, c + 8\,b\cos\left(\frac{1}{2}\kappa\right) a^2 \sin(\delta 2)\, c$$

$$-8\,b\cos\left(\frac{1}{2}\kappa\right)^3 a^2\sin(\delta 2)\,c\Big) \Big/ \Big(-16\,b^2\cos\left(\frac{1}{2}\kappa\right)^2 a^2 + 16\,b^2\cos\left(\frac{1}{2}\kappa\right)^4 a^2 + 4\,c^2\,b\,a - 4\,c^2\,b\,a\cos\left(\frac{1}{2}\kappa\right)^2 - 4\,c^2\,b\,a\cos(\delta 2)^2\Big)$$

The position constraint for S12

```
> algeq3:=t12/2-dotprod(col(Aline,1),evalm(Apoint-Bpoint),orthogonal);
```

$$algeq3 := \frac{1}{2}t12 - \cos(\delta 1)\left(-b\cos\left(\frac{1}{2}\kappa\right) + a\cos\left(\frac{1}{2}\kappa\right)\right) - \sin(\delta 1)\left(b\sin\left(\frac{1}{2}\kappa\right) + a\sin\left(\frac{1}{2}\kappa\right)\right)$$

The position constraint for S13

```
> algeq4:=t13/2-dotprod(col(Bline,1),evalm(Apoint-Bpoint),orthogonal);
```

$$algeq4 := \frac{1}{2}t13 - \cos(\delta 2)\left(-b\cos\left(\frac{1}{2}\kappa\right) + a\cos\left(\frac{1}{2}\kappa\right)\right) - \sin(\delta 2)\left(b\sin\left(\frac{1}{2}\kappa\right) + a\sin\left(\frac{1}{2}\kappa\right)\right)$$

```
>
```

Solve (symbolic)

Numeric values for the axes S12, S13. Numeric solution

Numeric values

Values in Tsai-Roth

```
> #Solutions
> rothw1:=vector(3,[0.3649,0.4520,0.8140]);
```

$$rothw1 := [.3649, .4520, .8140]$$

```
> rothwp1:=vector(3,[-0.9388,0.7580,0.7793]);
```

$$rothwp1 := [-.9388, .7580, .7793]$$

```
> Rothw1:=concat(rothw1,crossprod(rothwp1,rothw1));
```

$$Rothw1 := \begin{bmatrix} .3649 & .26476840 \\ .4520 & 1.04854977 \\ .8140 & -.70093180 \end{bmatrix}$$

```
> rothw2:=vector(3,[-0.5960,0.3593,-0.7181]);
```

$$rothw2 := [-.5960, .3593, -.7181]$$

```
> rothwp2:=vector(3,[0.8460,1.4034,-0.3793]);
```

$$rothwp2 := [.8460, 1.4034, -.3793]$$

```
> Rothw2:=concat(rothw2,crossprod(rothwp2,rothw2));
```

$$Rothw2 := \begin{bmatrix} -.5960 & -.87149905 \\ .3593 & .83357540 \\ -.7181 & 1.14039420 \end{bmatrix}$$

```
> rothg1:=vector(3,[0.5960,0.3593,0.7181]);
```

$$rothg1 := [.5960, .3593, .7181]$$

```
> rothwg1:=vector(3,[-0.8460,1.4034,0.3793]);
```

$$rothwg1 := [-.8460, 1.4034, .3793]$$

```
> Rothg1:=concat(rothg1,crossprod(rothwg1,rothg1));
```

$$Rothg1 := \begin{bmatrix} .5960 & .87149905 \\ .3593 & .83357540 \\ .7181 & -1.14039420 \end{bmatrix}$$

```
> rothg2:=vector(3,[-0.3649,0.4520,-0.8140]);
```

$$rothg2 := [-.3649, .4520, -.8140]$$

```
> rothwg2:=vector(3,[0.9388,0.7580,-0.7793]);
```

$$rothwg2 := [.9388, .7580, -.7793]$$

```
> Rothg2:=concat(rothg2,crossprod(rothwg2,rothg2));
```

$$Rothg2 := \begin{bmatrix} -.3649 & -.26476840 \\ .4520 & 1.04854977 \\ -.8140 & .70093180 \end{bmatrix}$$

```
> #Initial screws
> rS12:=matrix(3,2,[0,0,0,0,1,0]);
```

$$rS12 := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

```
> rS13:=evalf(concat(vector(3,[sin(30*Pi/180),0,cos(30*Pi/180)]),crosspr
  od(vector(3,[0,1,0]),vector(3,[sin(30*Pi/180),0,cos(30*Pi/180)]))));
```

$$rS13 := \begin{bmatrix} .5000000000 & .8660254040 \\ 0 & 0 \\ .8660254040 & -.5000000000 \end{bmatrix}$$

```
> rrot12:=evalf(40*Pi/180);
```

$$rrot12 := .6981317008$$

```
> rrot13:=evalf(70*Pi/180);
```

$$rrot13 := 1.221730477$$

```
> rtrans12:=0.8;
```

$$rtrans12 := .8$$

```
> rtrans13:=0.6;
```

$$rtrans13 := .6$$

Find principal axes (A, sigma, zoffset, pitches)

```
> distangle:=screwdot(rS12,rS13);
```

$$distangle := [\ .8660254040,\ -.5000000000\ ]$$

```
> rdist:=-distangle[2]/sqrt(1-distangle[1]^2);
```

$$rdist := 1.000000001$$

```
> rangle:=arctan(-distangle[2]/rdist,distangle[1]);
```

$$rangle := .5235987751$$

```
> evalf(rangle*180/Pi);
```

$$29.99999996$$

```
> rpitch12:=(rtrans12/2)/(tan(rrot12/2));
```

$$rpitch12 := 1.098990968$$

```
> rpitch13:=(rtrans13/2)/(tan(rrot13/2));
```

$$rpitch13 := .4284444017$$

```
> rZoffset:=(1/2)*(rdist-(rpitch13-rpitch12)*cos(rangle)/sin(rangle));
```

$$rZoffset := 1.080710362$$

```
> rAradius:=sqrt(rdist^2+(rpitch13-rpitch12)^2)/(2*sin(rangle));
```

$$rAradius := 1.204006936$$

```
> rsigma:=arctan((rdist*sin(rangle)-(rpitch13-rpitch12)*cos(rangle))/2,(
  (rpitch13-rpitch12)*sin(rangle)+rdist*cos(rangle))/2);
```

$$rsigma := 1.114282655$$

```
> #the principal axes are at half the distance and in the
  maximum/minimum pitch.
> rhx:=rpitch12+rAradius*cos(rsigma)-rAradius;
```

$$rhx := .425736154$$

```
> rhy:=rpitch12+rAradius*cos(rsigma)+rAradius;
```

$$rhy := 2.833750026$$

```
> rzx:=rAradius*sin(rsigma);
```

$$rzx := 1.080710362$$

```
> #rzx must be equal to rZoffset, and it is.
> #The principal axes are located at a distance rzx and rotated rsigma/2
  from the original reference axes, that are s12 and the perpendicular
  between s12 and s13.
```

Identify who is HX and who is HY

```
> angletoX:=1/2*(arccos((-rhx+rpitch12+rAradius*cos(rsigma))/rAradius)+r
  sigma);
```

$$angletoX := .5571413275$$

```
> evalf(angletoX*180/Pi);
```

$$31.92184667$$

```
> evalf(rsigma*90/Pi);
```

$$31.92184667$$

This proves HX is located at sigma/2 positive from S12

```
> angletoY:=1/2*(arccos((-rhy+rpitch12+rAradius*cos(rsigma))/rAradius)+r
  sigma);
```

$$angletoY := 2.127937655$$

```
> evalf(angletoY*180/Pi);
```

$$121.9218467$$

And HY at sigma/2+90

```
>
>
```

Write transformation A, from original axes to principal axes.

```
> #Transformation matrix
> Mtrans:=matrix(4,4,[sin(rsigma/2),0,cos(rsigma/2),0,cos(rsigma/2),0,-s
  in(rsigma/2),0,0,1,0,-rzx,0,0,0,1]);
```

$$Mtrans := \begin{bmatrix} .5287620059 & 0 & .8487701344 & 0 \\ .8487701344 & 0 & -.5287620059 & 0 \\ 0 & 1 & 0 & -1.080710362 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
> #############
  # SCREWTRANS
  #############
  # Routine to transform coordinates for screws.
  # Given the transformation matrix, multiplies by the dual vector
  #############
  #
  screwtrans:=proc(M,S)
  local S2,d,A,w,v,wa,va,ans;
  S2:=format(S);
  w:=col(S2,1);
  v:=col(S2,2);
  A:=submatrix(M,1..3,1..3);
  d:=vector(3,[M[1,4],M[2,4],M[3,4]]);
  wa:=evalm(A &* w);
  va:=evalm(crossprod(d,evalm(A &* w))+A &* v);
  ans:=concat(wa,va);
  eval(ans)
  end:
  #
```

```
> Z:=matrix(3,2,[0,0,0,0,1,0]); X:=matrix(3,2,[1,0,0,0,0,0]);
```

$$Z := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$X := \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

```
> rcheck:=evalm(duvemult(Z,vector(2,[cos(rsigma/2),-rZoffset*sin(rsigma/
  2)]))+duvemult(X,vector(2,[sin(rsigma/2),rZoffset*cos(rsigma/2)])));
```

$$rcheck := \begin{bmatrix} .5287620059 & .9172746792 \\ 0 & 0 \\ .8487701344 & -.5714385788 \end{bmatrix}$$

```
> screwtrans(Mtrans,rcheck);
```

$$\begin{bmatrix} .9999999999 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

```
> rcheck2:=evalm(duvemult(Z,vector(2,[cos((rsigma+Pi)/2),-rZoffset*sin((
  rsigma+Pi)/2)]))+duvemult(X,vector(2,[sin((rsigma+Pi)/2),rZoffset*cos(
  (rsigma+Pi)/2)])));
```

$$rcheck2 := \begin{bmatrix} .8487701344 & -.5714385788 \\ 0 & 0 \\ -.5287620059 & -.9172746792 \end{bmatrix}$$

```
> screwtrans(Mtrans,rcheck2);
```

$$\begin{bmatrix} 0 & 0 \\ .9999999999 & 0 \\ 0 & 0 \end{bmatrix}$$

```
> #Transforming the displacement screws
> evalm(rS12);
```

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

```
> rpS12:=screwtrans(Mtrans,rS12);
```

$$rpS12 := \begin{bmatrix} .8487701344 & -.5714385788 \\ -.5287620059 & -.9172746792 \\ 0 & 0 \end{bmatrix}$$

```
> rpS13:=screwtrans(Mtrans,rS13);
```

$$rpS13 := \begin{bmatrix} .9994375015 & -.00270672389 \\ -.0335362626 & -.0806649625 \\ 0 & 0 \end{bmatrix}$$

```
> isline(rpS12);
```

$$true$$

```
> dotprod(col(rpS13,1),col(rpS13,2),orthogonal);
```

$$.3\ 10^{-11}$$

```
> #Yupi!!!
```

Values of d, delta for the numeric solution

```
>
> cosdelta1:=rpS12[1,1];
```

$$cosdelta1 := .8487701344$$

```
> sindelta1:=rpS12[2,1];
```

$$sindelta1 := -.5287620059$$

```
> deltaone:=evalf(180*arctan(sindelta1,cosdelta1)/Pi);
```

$$deltaone := -31.92184667$$

```
> de1:=-rpS12[1,2]/sindelta1;
```

$$de1 := -1.080710362$$

```
> cosdelta2:=rpS13[1,1];
```

$$cosdelta2 := .9994375015$$

```
> sindelta2:=rpS13[2,1];
```

$$sindelta2 := -.0335362626$$

```
> deltatwo:=evalf(180*arctan(sindelta2,cosdelta2)/Pi);
```

$$deltatwo := -1.921846665$$

```
> de2:=-rpS13[1,2]/sindelta2;
```

$$de2 := -.08071036186$$

Numeric solution

```
> values1:=subs(d1=de1,d2=de2,delta1=deltaone*Pi/180,delta2=deltatwo*Pi/180
  ,t12=0.8,t13=0.6,theta12=40*Pi/180,theta13=70*Pi/180,algeq1);
```

$$values1 := \tan\left(\frac{1}{9}\pi\right) + \left(8\,c\,b^2\sin(-.1773435926\,\pi)\cos\left(\frac{1}{2}\kappa\right)a - 8\,c\,b^2\sin(-.1773435926\,\pi)\cos\left(\frac{1}{2}\kappa\right)^3 a - 8\,c\,b^2\cos\left(\frac{1}{2}\kappa\right)^2\cos(-.1773435926\,\pi)\,a\sin\left(\frac{1}{2}\kappa\right) + 8\,b\cos\left(\frac{1}{2}\kappa\right)^2 a^2\sin\left(\frac{1}{2}\kappa\right)\cos(-.1773435926\,\pi)\,c + 8\,b\cos\left(\frac{1}{2}\kappa\right)a^2\sin(-.1773435926\,\pi)\,c - 8\,b\cos\left(\frac{1}{2}\kappa\right)^3 a^2\sin(-.1773435926\,\pi)\,c\right) \Big/ \left(-16\,b^2\cos\left(\frac{1}{2}\kappa\right)^2 a^2 + 16\,b^2\cos\left(\frac{1}{2}\kappa\right)^4 a^2 + 4\,c^2\,b\,a - 4\,c^2\,b\,a\cos\left(\frac{1}{2}\kappa\right)^2 - 4\,c^2\,b\,a\cos(-.1773435926\,\pi)^2\right)$$

```
> values2:=subs(d1=de1,d2=de2,delta1=deltaone*Pi/180,delta2=deltatwo*Pi/180
  ,t12=0.8,t13=0.6,theta12=40*Pi/180,theta13=70*Pi/180,algeq2);
```

$$values2 := \tan\left(\frac{7}{36}\pi\right) + \left(8\,c\,b^2\sin(-.01067692592\,\pi)\cos\left(\frac{1}{2}\kappa\right)a - 8\,c\,b^2\sin(-.01067692592\,\pi)\cos\left(\frac{1}{2}\kappa\right)^3 a - 8\,c\,b^2\cos\left(\frac{1}{2}\kappa\right)^2\cos(-.01067692592\,\pi)\,a\sin\left(\frac{1}{2}\kappa\right) + 8\,b\cos\left(\frac{1}{2}\kappa\right)^2 a^2\sin\left(\frac{1}{2}\kappa\right)\cos(-.01067692592\,\pi)\,c + 8\,b\cos\left(\frac{1}{2}\kappa\right)a^2\sin(-.01067692592\,\pi)\,c - 8\,b\cos\left(\frac{1}{2}\kappa\right)^3 a^2\sin(-.01067692592\,\pi)\,c\right) \Big/ \left(-16\,b^2\cos\left(\frac{1}{2}\kappa\right)^2 a^2 + 16\,b^2\cos\left(\frac{1}{2}\kappa\right)^4 a^2 + 4\,c^2\,b\,a - 4\,c^2\,b\,a\cos\left(\frac{1}{2}\kappa\right)^2 - 4\,c^2\,b\,a\cos(-.01067692592\,\pi)^2\right)$$

```
> values3:=subs(d1=de1,d2=de2,delta1=deltaone*Pi/180,delta2=deltatwo*Pi/180
  ,t12=0.8,t13=0.6,theta12=40*Pi/180,theta13=70*Pi/180,algeq3);
```

$$values3 := .4000000000 - \cos(-.1773435926\,\pi)\left(-b\cos\left(\frac{1}{2}\kappa\right) + a\cos\left(\frac{1}{2}\kappa\right)\right) - \sin(-.1773435926\,\pi)\left(b\sin\left(\frac{1}{2}\kappa\right) + a\sin\left(\frac{1}{2}\kappa\right)\right)$$

```
> values4:=subs(d1=de1,d2=de2,delta1=deltaone*Pi/180,delta2=deltatwo*Pi/180
  ,t12=0.8,t13=0.6,theta12=40*Pi/180,theta13=70*Pi/180,algeq4);
```

$$values4 := .3000000000 - \cos(-.01067692592\,\pi)\left(-b\cos\left(\frac{1}{2}\kappa\right) + a\cos\left(\frac{1}{2}\kappa\right)\right) - \sin(-.01067692592\,\pi)\left(b\sin\left(\frac{1}{2}\kappa\right) + a\sin\left(\frac{1}{2}\kappa\right)\right)$$

```
> fsolve({values1,values2,values3,values4},{a,b,c,kappa});
```

$$\{b = .9271547677,\ c = .6453985164,\ \kappa = 3.412931041,\ a = -1.220134870\}$$

These two sets of solutions are obtained changing the sign of t1i/2 and theta1i/2. They correspond to both pair of dyads of the mechanism

With pitch equal to disp over tan

```
> afinal:=-1.220134870;bfinal:=.9271547677;cfinal:=.6453985164;kappafinal:=
  3.412931041;
```

$$afinal := -1.220134870$$
$$bfinal := .9271547677$$
$$cfinal := .6453985164$$
$$kappafinal := 3.412931041$$

```
> Gnumer:=evalf(subs(a=afinal,b=bfinal,c=cfinal,kappa=kappafinal,axiscrew(G
  screw)));
```

$$Gnumer := \begin{bmatrix} .8838278702 & -.5853506912 \\ -.1206493541 & -.3598007854 \\ .4519867580 & 1.048569486 \end{bmatrix}$$

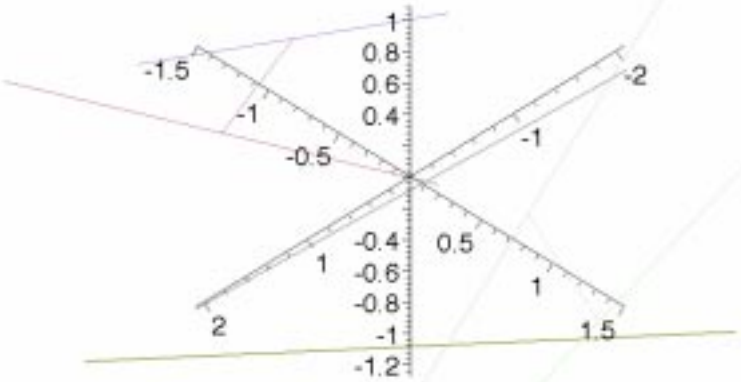
```
> Wnumer:=evalf(subs(a=afinal,b=bfinal,c=cfinal,kappa=kappafinal,axiscrew(W
  screw)));
```

$$Wnumer := \begin{bmatrix} .9246409849 & -.3708108723 \\ .1262206607 & .3434393781 \\ .3593151722 & .8335795652 \end{bmatrix}$$

```
> ploty1:=lineplot3(axiscrew(Gnumer),axiscrew(Wnumer));
Share Library:  dualvectors
Author: Perez,Alba.
Description:  Performs dual vectors and numbers operations.
```

$$ploty1 := \text{PLOT3D}(\text{CURVES}([[-1.602628369, -.9676243958, -1.226672776], [1.932683111, -1.450221812, .5812742565]]), \text{CURVES}([[1.723881146, -.6661938616, 1.041329602], [-1.974682794, -1.171076504, -.3959310868]])$$

```
    , CURVES(
    [[-.1254008243, -.9186351830, .3226992576], [.1650273710, -1.208923104, -.3226992595]],
    LINESTYLE(3)), AXESSTYLE(FRAME))
> ploty2:=lineplot3(rpS12,rpS13);
Share Library:  dualvectors
Author: Perez,Alba.
Description:  Performs dual vectors and numbers operations.
ploty2 := PLOT3D(CURVES(
    [[1.697540269, -1.057524012, -1.080710362], [-1.697540269, 1.057524012, -1.080710362]]),
    CURVES(
    [[-1.998875003, .0670725252, -.08071036198], [1.998875003, -.0670725252, -.08071036198]]),
    CURVES([[0, 0, -.08071036198], [0, 0, -1.080710362]], LINESTYLE(3)), AXESSTYLE(FRAME))
> display1:=display(ploty2,thickness=2);
display1 := PLOT3D(CURVES(
    [[1.697540269, -1.057524012, -1.080710362], [-1.697540269, 1.057524012, -1.080710362]]),
    CURVES(
    [[-1.998875003, .0670725252, -.08071036198], [1.998875003, -.0670725252, -.08071036198]]),
    CURVES([[0, 0, -.08071036198], [0, 0, -1.080710362]], LINESTYLE(3)), THICKNESS(2),
    AXESSTYLE(FRAME))
> ploty3:=lineplot3(axiscrew(screwtrans(Mtrans,Rothg1)),axiscrew(screwtrans
  (Mtrans,Rothw1)));
Share Library:  dualvectors
Author: Perez,Alba.
Description:  Performs dual vectors and numbers operations.
ploty3 := PLOT3D(CURVES(
    [[-1.974395247, -1.170905826, -.3957988870], [1.724217509, -.6662487904, 1.041415413]]),
    CURVES([[1.932953961, -1.450311897, .5814025948], [-1.602330685, -.9675402486, -1.226550390]])
    , CURVES(
    [[.1653116379, -1.208926073, -.3225738974], [-.1250888687, -.9185773084, .3228082630]],
    LINESTYLE(3)), AXESSTYLE(FRAME))
> ploty4:=lineplot3(axiscrew(screwtrans(Mtrans,Rothg2)),axiscrew(screwtrans
  (Mtrans,Rothw2)));
Share Library:  dualvectors
Author: Perez,Alba.
Description:  Performs dual vectors and numbers operations.
ploty4 := PLOT3D(
    CURVES([[-1.932953961, 1.450311897, .5814025948], [1.602330685, .9675402486, -1.226550390]]),
    CURVES([[-1.724217509, .6662487904, 1.041415413], [1.974395247, 1.170905826, -.3957988870]]),
    CURVES([[.1250888687, .9185773084, .3228082630], [-.1653116379, 1.208926073, -.3225738974]],
    LINESTYLE(3)), AXESSTYLE(FRAME))
> display(ploty3,ploty4,display1,ploty1);
```


1
0.8
0.6
0.4
-1.5
-1
-0.5
-2
-1
-0.4
-0.6
-0.8
-1
-1.2
0.5
1
1
2
1.5

>